

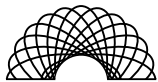
A Short History of Rocks: or, How to Invent Quantum Computing

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ABSTRACT

This essay gives a short, informal account of the development of digital logic from the Pleistocene to the Manhattan Project, the introduction of reversible circuits, and Richard Feynman’s allied proposal for quantum computing. We argue that Feynman’s state-based analogy is not the only way to arrive at quantum computing, nor indeed the simplest. To illustrate, we imagine an alternate timeline in which John von Neumann skipped Operation Crossroads to debug a military computer, got tickled by the problem, and discovered a completely different picture of quantum computing—in 1946.

Feynman suggested we “quantize” state, and turn classically reversible circuits into quantum reversible, unitary ones. In contrast, we speculate that von Neumann, with his background in functional analysis and quantum logic, would seek to “quantize” the operators of Boolean algebra, and with tools made available in 1946 could successfully do so. This leads to a simpler, more flexible circuit calculus and beautiful parallels to classical logic, as we detail in a forthcoming companion paper.



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o *Overview*

This essay gives a short history of human-rock interactions, arguing that, far from tricking rocks into doing math, they tricked us; or rather, the trickery is mutual and ongoing. We support this thesis by example, progressing through number systems, binary mysticism, Boolean logic, digital circuits, and large-scale programmable architectures, drawing attention at each juncture to the collaborative role played by our igneous friends. We conclude the first half with quantum computing, in some sense the apotheosis of this mutual trickery.

History abhors a linear narrative; in our case, the linear narrative is undone by the Manhattan Project, which led to a long chill between the physicists who worked on the bomb—including Richard Feynmann—and the ductile rocks that helped turn an equation as beautiful as $E = mc^2$ into Little Boy and Fat Man. The physicists left rocks behind and turned to theory. Although the rocks tried to teach us quantum, for a long time, we didn't listen; when eventually, reluctantly, we did, the transmission was garbled.

In the second half of this essay, we try to clarify what the rocks might have meant, using the clues scattered throughout history and some generous counterfactual license. It may be that, in a branch of the wavefunction not so far from this one, John von Neumann invented quantum computing 35 years before Feynman, using functional analysis to generalize Shannon's algebra of circuits. We give a fuller development of this formalism elsewhere, but hope that in the mean time, the reader is a little more attuned to the whispering presence of the inanimate world.

ACKNOWLEDGMENTS

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1	<i>The other calculus</i>	3
2	<i>Computing at scale</i>	5
3	<i>A fork in the roadmap</i>	9
4	<i>Through the looking glass</i>	13
5	<i>Postscript</i>	15

1 *The other calculus*

The development of mathematics and technology is deeply tied to our interaction with rocks. During the Pleistocene, humans discovered the number 1 and began to count in unary on their fingers, with pebbles, or by repetitive scratches onto stone or slate. We can think of these techniques as a unary programming language (a formal system for representing algorithms) and the rocks as an early form of computer (a means of carrying those algorithms out).

The transition from hands to tally marks is instructive. Tallies were invented because we ran out of fingers; each round of five marks effectively supplies an extra hand. In fact, the diagonal slash for grouping tallies is an *abstraction* of the hand, an abstraction forced by computational necessity. As civilization grew, numbers grew with it, and our number systems—with associated abstractions and algorithms—had to keep up. Tally marks became sign-value systems (like Roman numerals, or Sumerian/Babylonian cuneiform numerals below 60) and then place-value systems (like Babylonian sexagesimals). The sequence of hierarchizing macros in between was not so different from the slash which replaced the hand.

During this evolution, stones were supplanted by more portable computers like clay, papyrus and parchment. The rocks—our steadfast, inanimate friends—would eventually make a comeback, but not before the wheel of fortune had revolved a few times. Place value systems require a way to indicate place. While the Babylonians sidestepped the problem, hoping context or an empty space would make it clear, Indian mathematicians introduced the symbol “●” as a (literal) placeholder. This eventually morphed into the digit “0”. In contrast to lumbering bases like sixty (probably chosen for its compositeness) and ten (once again, replacing hands), it was now possible to build a number system from 0 and 1 alone.

It took people a while to catch on. A full 1500 years after 0 first appeared, German polymath GOTTFRIED WILHELM LEIBNIZ began to noodle around with binary arithmetic. Leibniz was a Sinophile (among many other things) and learned that the “hexagrams” of the Chinese *Book of Changes* corresponded to binary labels.¹ He took this parallel—across a vast interval of time, language, and culture—as proof of the underlying universality of human thought.

This universality would become one of his central preoccupations. First, it suggested the possibility of a universal language or *characteristica universalis* (“general characteristic”), which he envisioned as



Unary programming in silico, aka tallying. Tallies exist because we ran out of fingers.



Ontogeny recapitulates phylogeny in Babylonian numerals. Left. A unary tally for 3. Middle. Sign-value representation of 11. Right. 1277 in place-value notation.



Gottfried Leibniz (1646–1716). Mathematician, Sinophile, and peruke enthusiast.

¹ To be clear, these did not constitute a place value system.

a fantastically expressive pictorial script “by which all concepts and things [could] be put into beautiful order,”² but which would be precise enough to reason about mathematically. To actually perform this reasoning, Leibniz introduced another powerful abstraction: the *calculus ratiocinator* (“reasoning calculator”), a “general algebra in which all truths of reason would be reduced to a kind of calculus.”³ Though less well-known than Leibniz’s work on infinitesimal calculus, the *characteristica* and the *ratiocinator* are no less important. They laid the conceptual foundations of modern digital programming (*characteristica*) and computation (*ratiocinator*).

He didn’t live to see either utopian project realized. But a century after Leibniz’s death, the wife of a downtrodden Lincolnshire cobbler gave birth to a son. The cobbler was more enthusiastic about science than shoes, and his child—GEORGE BOOLE—would leave school early to pick up the slack, teach himself mathematics in his spare time, and blaze the trail to a professorship at Queen’s University. He would also build the “general algebra” that Leibniz had dreamt of, and from the binary components of the *characteristica*, no less.

Boole’s crucial insight was that logic could be *algebraized*. If we identify 1 with TRUE and 0 with FALSE, then basic logical operations like AND (\wedge), OR (\vee), and NOT (\neg) become algebraic:

$$x \wedge y = x \cdot y, \quad x \vee y = x + y - xy, \quad \neg x = 1 - x,$$

where x, y are variables representing propositions, and the operations on the right are ordinary arithmetic. These observations are surprisingly powerful. For instance, the algebraic fact that

$$1 - xy = (1 - x) + (1 - y) - (1 - x)(1 - y)$$

is equivalent to DE MORGAN’S LAW:

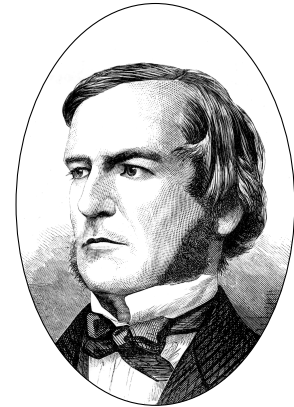
$$\neg(x \wedge y) = \neg x \vee \neg y.$$

This structure, called a **BOOLEAN ALGEBRA**, indeed reduces the “truths of reason” to a remarkably simple calculus.⁴

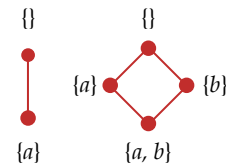
Let’s recap. We started counting 1s, ran out of fingers, drew on rocks, ran out of room, invented sign value, place value, then 0, and ported it all to lighter computers. That was good for a while, until a strange little man with a wig started counting in 1s and 0s, noticed the same in a 2500-year old Chinese divination manual, figured that made a good case for a universal calculus, which a shoemaker’s son cobbled together, from 1s and 0s, 150 years later. The scene was now set for the return of the rocks.

² “On the General Characteristic” (1679).

³ Letter to Nicolas Remond (1714).



George Boole (1815–1864). Son of a Lincolnshire cobbler, heir of Leibniz.



A concrete realization of Boolean algebra using sets. Logical operations correspond to set operations: union (\vee), intersection (\wedge), and complement (\neg).

⁴ See *The Mathematical Analysis of Logic* (1847) and *An Investigation of the Laws of Thought* (1854).

2 Computing at scale

CLAUDE ELWOOD SHANNON encountered the work of George Boole as an undergraduate doing dual degrees in mathematics and electrical engineering. Born in the dusty crossroads of Gaylord, Michigan, Shannon enjoyed puzzles, games, and taking apart old machinery only to reassemble it in surprising new ways. During his masters at MIT, he was tasked with studying the “differential analyzer”. The analyzer—a steampunk vision of electromechanical relays and ad hoc circuitry—was built to solve differential equations (ironically one of Boole’s main interests as a mathematician) and pioneered by the brother of Lord Kelvin (one of Boole’s close friends). Shannon’s fitting tribute to Boole was to deconstruct the messy circuitry of the analyzer and systematically resynthesize it with Boolean algebra. In the process, he invented modern digital circuitry.

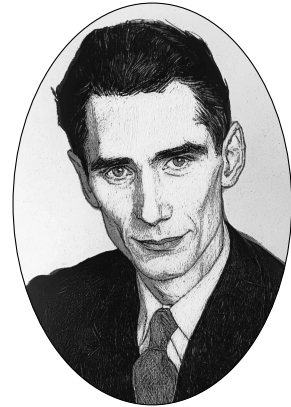
After a postdoc at the Institute for Advanced Study (IAS)—where Einstein called him “a brilliant, brilliant man”⁵—Shannon hopped over the river from New Jersey to Bell Labs, then based in Manhattan. A gaunt, courteous wizard who kept to himself (though he sometimes roamed the Labs by unicycle), he would abundantly justify Einstein’s praise. Shannon went on to single-handedly create modern cryptography, information and communication theory,⁶ establishing that any contentful message could be converted into a stream of 1s and 0s to be processed on the Boolean circuits he had devised as a graduate student. This created a bridge from language to computation, or in Leibnizian terms, the *characteristica universalis* to the *calculus ratiocinator*. Shannon’s “A Symbolic Analysis of Relay and Switching Circuits” may be the greatest masters thesis ever written, but the sequel was better than the original.

A tally is a line scratched in rock; a circuit is lines drawn in metal. Though *ratiocinators* would grow ever larger and more sophisticated, becoming the smartphones and laptops and high-performance GPUs we have today, many layers of abstraction down is a “general algebra” of 1s and 0s, playing across the metal in bursts of current. It took a few thousand years, but we tricked rocks into doing binary.

While at the IAS, Shannon crossed paths with JOHN VON NEUMANN, the Hungarian-American mathematician and youngest faculty member at the Institute. The story goes that, by 1940, Shannon had already struck upon his famous formula

$$S[p] = - \sum_{i=1}^n p_i \log_2 p_i = \mathbb{E}_p[-\log_2 p]$$

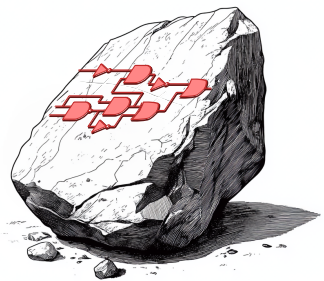
for the amount of information contained in a probability distribution $p = (p_i)$. But he struggled with the name, hesitating between “infor-



Claude Shannon (1916–2001). The quiet magician who tricked rocks into thinking.

⁵ *The Idea Factory* (2013), Jon Gertner.

⁶ See “A Mathematical Theory of Cryptography” (1945), “A Mathematical Theory of Communication” (1948).



Binary programming in silico, aka digital circuits. These exist because we ran out of fingers, and found a symbol for “ran out”.

mation” and “uncertainty”. Von Neumann offered a third option:⁷

You should call it *entropy*, for two reasons. In the first place your uncertainty function has been used in statistical mechanics under that name. In the second place, and more importantly, no one knows what entropy really is, so in a debate you will always have the advantage.⁸

It’s unlikely Shannon needed the advantage, but the name stuck.

Von Neumann was, in many ways, the opposite of Shannon. Boisterous, earthy, outgoing, von Neumann hailed from Budapest, the glittering capital of the Austro-Hungarian empire; Shannon was a wallflower from the backwaters of the Midwest. During WWII, Shannon stayed at Bell Labs, hoping to quietly avoid the draft; von Neumann signed up immediately and was rejected due to age, not zeal. And where Shannon was thorough, methodical and focused, sometimes letting a problem steep for years, von Neumann was broad and almost inhumanly quick, cutting a dazzling swathe through 20th century mathematics both pure and applied. As Hans Bethe wrote⁹

I have sometimes wondered whether a brain like von Neumann’s does not indicate a species superior to that of man.

Shannon called him the smartest man he had ever met. But despite their differences, the two shared a yen for applied problems (both had degrees in engineering) and would, increasingly, spend their time thinking about computers, conduits of the entropy that Shannon had fathered and von Neumann baptized.

While Shannon laid low, the war took von Neumann to Los Alamos, where he worked with characteristic vigour on the science of blowing things up. Part of this science was numerical, and involved a new toy from the US Ballistic Research Laboratory: the ELECTRONIC NUMERICAL INTEGRATOR AND COMPUTER (ENIAC). ENIAC was a little like the differential analyzer Shannon had studied, but larger, faster, and most importantly, *programmable*. You could run different programs simply by swapping out punch cards. Von Neumann would develop some of the first, very primitive, programming languages in order to tell ENIAC how to run thermonuclear simulations.

Programming on punch cards is a bit like counting on fingers; it only works for small problems. The architects of ENIAC (JOHN MAUCHLY and J. PRESPEER ECKERT) realized they needed a way to store programs and data, and Eckert invented a clever memory unit based on pinging signals through mercury. This was one of a number of innovations bundled into ENIAC’s successor, the ELECTRONIC DISCRETE VARIABLE AUTOMATIC COMPUTER (EDVAC), on which Mauchly and Eckert gave lectures in 1945. Von Neumann took polished, comprehensive notes, peppered with original insights, which an incautious colleague began to circulate; the notes went

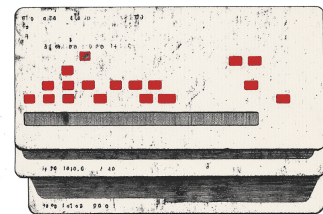
⁷ “Energy and information” (1971), Myron Tribus and Edward McIrvine.

⁸ “Torsor” may have been chosen according to the same guiding principle.



Here’s Johnny! (1903–1957) A genius with a strange urge to blow things up.

⁹ “Passing of a Great Mind” (1957), Clay Blair Jr.



ENIAC punch cards. As tallies led to sign value, the inconvenience of punch cards led to the “von Neumann” architecture.

viral, quashing Mauchly and Eckert's patent claims and leading to the permanent misattribution "von Neumann architecture" for EDVAC's design scheme. Von Neumann's reputation preceded not only himself, but his colleagues as well.

Whatever the precise division of credit, von Neumann was central to the early history of computing at scale, creating the first protocols for talking to ENIAC/EDVAC, its first applications, its first bespoke algorithms (merge sort and Monte Carlo approximation with Stan Ulam), and aspects of the first integrated, stored-program architecture. After the war, he would increasingly focus on methods for large-scale numerical and scientific computing, including the first climate-modeling software, run on ENIAC.¹⁰ We can only wonder what else his marvelous organic brain might have achieved in concert with the electronic brains at his disposal. Von Neumann died from cancer in 1957, probably caused by wartime radiation exposure. He left his mark on the bomb; it left its mark on him.

The Manhattan Project also left subtler marks. Von Neumann's assistant on the ENIAC simulations—handling punch card operations—was a bright young theoretician called RICHARD FEYNMAN. Feynman would later win a Nobel Prize for his work on quantum electrodynamics, and become legendary for his originality, intuition, and goofy, homespun charm. In his autobiography,¹¹ he makes his time at the Manhattan Project sound like a sequence of wise-cracking, safe-cracking hijinks. But as historian Cathryn Carson soberly observes:¹²

The reality was somewhat grimmer: the coded letters, for instance, were to his wife, his highschool sweetheart, dying of tuberculosis in a cheap sanatorium outside Albuquerque. The real lessons Feynman learned at Los Alamos [were] how to hide his feelings behind a brash facade and how to excise unwelcome memories.

Feynman was perhaps less happy to estimate a death toll, or the optimal height to detonate a bomb, than the hawkish von Neumann.

Feynman may have distanced himself emotionally by becoming a "curious character," the bongo-playing beatnik and hero of every anecdote. But he also distanced himself scientifically. In contrast to von Neumann, the champion of large-scale computation, Feynman would turn to the physics of the very small. His Nobel Prize-winning work made the leap from the quantum mechanics of point-like particles to spatially extended objects called *fields*, and thereby helped establish the framework of *quantum field theory*. Perhaps his most famous contribution was a graphical technique called *Feynman diagrams* for approximating the probability that one set of particles will collide and transform into another set.

Feynman's gift for the very small was not just theoretical. In 1959, he gave a prescient lecture to the American Physical Society called

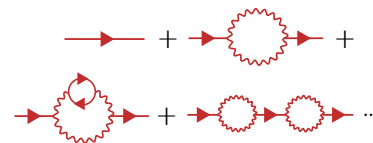
¹⁰ "Numerical Integration of the Barotropic Vorticity Equation" (1950), with Jule Charney and Ragnar Fjørtoft.



Richard P. Feynman (1918–1988). *The folk hero of the very small.*

¹¹ *Surely You're Joking, Mr. Feynman!* (1985).

¹² "An Eden after the Fall" (1993).



Feynman diagrams for an electron minding its own business (aka the electron propagator). The wiggles are virtual photons.

“There’s Plenty of Room at the Bottom”, with the general theme of tricking tiny rocks into doing our bidding. This more or less inspired the field of nanotechnology. One thought experiment—adapted, in fact, from von Neumann—was *scaled replication*, where a hierarchy of ever smaller robot hands is used to eventually build at the nanoscale. The lecture wasn’t appreciated until the 80s when experimental methods were finally up to the task of constructing molecular machines. One of Feynman’s challenges—to print the *Encyclopædia Britannica* on the head of a pin—was only cracked in 1985.

While getting the *Britannica* to dance on the head of a pin is a colourful Feynman-esque conceit, more intriguing was his brief mention of miniaturizing computers, fifteen years after his ENIAC tour of duty and ten years before the first microprocessor:

...there is plenty of room to make [computers] smaller. There is nothing that I can see in the physical laws that says the computer elements cannot be made enormously smaller than they are now. In fact, there may be certain advantages.

These “advantages” were left mostly unspecified, and Feynman moved on to other tasks—his Caltech lectures, the puzzles of partons, a new Lagrangian-based approach to quantum mechanics—that were more pressing and immediately soluble.

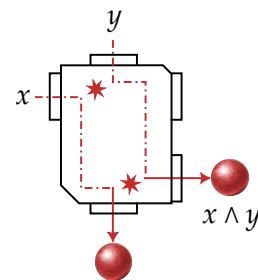
Feynman might never have returned to the problem were it not for a college dropout called EDWARD FREDKIN. Fredkin, a self-taught programmer, floated between consulting, industry, and sporadic faculty appointments at Carnegie Mellon and MIT. One such stint was as Director of Project MAC at MIT (a predecessor of CSAIL) from 1971–74; after three years he got bored, and decided to head to Caltech to spend time with Feynman, who he’d met in 1962 and found enjoyably provocative. They struck a deal. Fredkin would stay for a year and teach Feynman about computing; Feynman would teach him about quantum physics. Both were somewhat skeptical about what was on offer, but committed to learning.

It was a slow burn win-win. Fredkin successfully mastered quantum mechanics, but was unconvinced the universe could be fundamentally continuous (it didn’t compute!). He transformed that resistance into a successful research program for “digital physics”, where discrete objects like cellular automata were used to effectively mock up known physical laws. It also motivated him to explore *reversible* computation, since all microscopic laws are time-reversal invariant.¹³ The burn was slower for Feynman. He remained unsure that physics and computation could be usefully connected; maybe, after the bomb, he didn’t want to connect them.

Regardless, the two remained close, and in 1981, Fredkin invited the physicist to give the keynote at an MIT conference on physics



Edward Fredkin (1934–2023). Millionaire, MIT professor and college dropout. Take from that what you will.



AND gate for the reversible billiard-ball computer designed by Fredkin and Tomaso Toffoli. A ball represents 1; its absence, 0.

¹³ Technically, CPT-invariant, but we won’t split hairs. See “Conservative Logic” (1982), Fredkin and Toffoli.

and computation. It was going to be a lot of digital physics “guff” and Feynman was reluctant; he agreed, however, after Fredkin gave him *carte blanche* on the topic. Feynman opened with this warm and revealing tribute:¹⁴

The reason for doing this is something that I learned about from Ed Fredkin, and my entire interest in the subject has been inspired by him. It has to do with learning something about the possibilities of computers, and also something about possibilities in physics.

Feynman’s hour-long address would explore the possibilities posed by simulating the physics of the very small, propose a new type of machine called a *quantum computer* to address it, and kickstart a whole new field of computational science in the process.

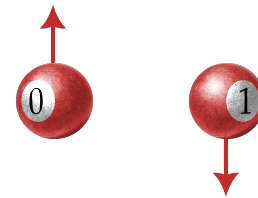
Feynman’s intuition was simple: it should be easier to imitate quantum physics with a computer running on quantum principles. He gave a heuristic argument to this effect, and outlined a scheme for universal quantum simulation using what he called spin- $\frac{1}{2}$ systems. These have two possible states, usually denoted $|1\rangle$ and $|0\rangle$, so they are the quantum analogue of a bit, also called a *qubit*. Though Feynman’s motivations were rather different from the assembled group, many early contributors to quantum computing—Charles Bennett, Norman Margolus, Tomaso Toffoli, and Fredkin himself—were present at the talk. The way we reason about quantum computing, using qubits, circuits, and reversible logic, bears their digital imprint.

It’s tempting (and indeed customary) to view the qubit as the natural endpoint of this tortuous back-and-forth between human and rock. We scratched in unary on rocks, etched binary in metal, then listened carefully and let metal teach us a new type of binary. Now we are scaling Feynman’s ladder in reverse and extrapolating hardware from the very small to the macroscopic, hoping to perform simulations more powerful than Feynman or von Neumann ever dreamed of. It’s a nice story, and it happens to be the reality we live in. But it didn’t have to be.

3 *A fork in the roadmap*

We can try to picture a different path. It starts around 3000 light years away, with middle-aged couple dancing slowly through space. T CORONAE BOREALIS (T CRB) is a binary system in the constellation Coronae Borealis and the figurative jewel in its crown, consisting of a red giant and a white dwarf gradually accreting material from its larger companion. Every 80 years or so, the white dwarf takes a giant slurp of charged matter and blazes into view; this *recurrent nova* briefly outshines most stars in the sky. It blazes not only in radiation,

¹⁴ “Simulating Physics with Computers” (1982). On the other side of the Iron Curtain, Yuri Manin independently had the same idea in 1980.



Spin- $\frac{1}{2}$ particles have two states: up $|0\rangle$ and down $|1\rangle$. Feynman also used absent/present, like the billiard balls.



Quantum binary in silico, aka quantum circuits. Quantum circuits exist because small-scale simulation is hard.

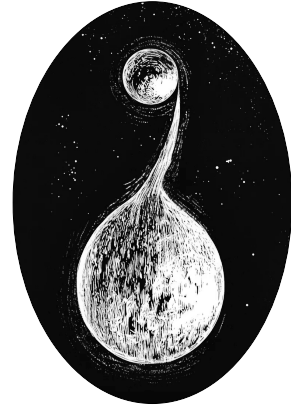
but sometimes lone protons, which are whipped around this stellar cyclotron and flung out at close to the speed of light. We call these protons cosmic rays.

When cosmic rays hit the atmosphere, they fragment into a cascade of secondary particles. This is a quantum process, described by Feynman diagrams; the effect of a cosmic ray depends on the fragments. In February 1946, T CrB went nova, with another luminosity bump in June. In some quantum fork of history, a cosmic ray broke up over Philadelphia, showered a bank of vacuum tubes with ionizing radiation, and knocked ENIAC offline a month before it was due to be handed over to the military. The operators knew it couldn't be background radiation—that struck one tube at a time—and Mauchly and Eckert, now running the Electronic Control Company, began to speculate about new kinds of systemic failure. An Ordnance Corps concerned about their strategically crucial, multi-million dollar investment requisitioned von Neumann from his efforts to design a new stored-program computer at the IAS.¹⁵

In July of 1946, Operation Crossroads¹⁶ would take place on Bikini Atoll, where some physicists would receive possibly lethal doses of radiation from the spectacular but mismanaged Baker test. In reality, von Neumann was one of these physicists; in the fork, he wasn't there. He was in Pennsylvania, where ENIAC's flipped tubes revealed a subtle directional gradient in ionization; from the time of failure, the gradient pointed to a spot in the sky with right ascension 16 hours and declination +26°. Then he was in Chicago, talking to his old collaborator Subrahmanyan Chandrasekhar about radiative transfer, then Pasadena to discuss novae with Walter Baade and cosmic showers with his old boss Oppenheimer. Finally, he flew back to New Jersey, where he argued with Stibitz and Hamming at Bell Labs and stayed up late playing chess with Shannon.

Von Neumann had immediately guessed the culprit was a cosmic ray. The astrophysicists confirmed this guess and offered a candidate, T CrB; Oppenheimer, adrift at Caltech, was happy to tease out the energetics of fragmentation. At Bell, Hamming and Stibitz would talk guardedly about error correction, while Shannon, with typical modesty, would outline a foundational perspective on noisy channels. Von Neumann took it all in, overlaid and interfered the conversations, and by the time he returned to Princeton had arrived at a novel conclusion: instead of *correcting* cosmic rays, perhaps they could *control* them? After all, they had created "artificial" rays for implosion imaging.¹⁷ Why not compute quantum with quantum?

In this version of reality, von Neumann was led to think about quantum computing more than thirty years before Feynman. Feynman had a genius for the miniscule; naturally, he wanted to use



T Coronae Borealis (1866–). The deus ex stellae of our alternate timeline.

¹⁵ See "Preliminary discussion of the logical design of an electronic computing instrument" (1946), Burks, Goldstine, and von Neumann.

¹⁶ When history hands you a name this perfect, you don't refuse.



The cosmic ray event at ENIAC that could have saved John von Neumann's life.

¹⁷ See "Flash radiography with 24 GeV/c protons" (2011), C. Morris et al.

computers to understand his small friends better, and turned to the techniques—states, transition amplitudes, diagrams, reversible circuits—he was familiar with. The result is quantum computing as we know it. Von Neumann, in contrast, was a near scale-invariant scientist. He studied stars and planetary-scale weather systems, macroscopic architectures, and long before the war, laid the mathematical foundations for quantum mechanics. His goals and techniques spanned many more orders of magnitude.

Let's start at the bottom. Von Neumann worked with DAVID HILBERT¹⁸ to define what we call *Hilbert space*. This is a vector space \mathcal{H} over the complex numbers \mathbb{C} , with an inner product $\langle \cdot, \cdot \rangle$ and closed with respect to the induced norm.¹⁹ A *state* is a unit length vector in this space. This is the usual arena of quantum mechanics and quantum computing. Ten years later, however, von Neumann had become skeptical of the Hilbert space formalism, writing²⁰

I would like to make a confession which may seem immoral: I do not believe absolutely in Hilbert space any more. . . . Because: (1) The vectors ought to represent the physical states, but they do it redundantly, up to a complex factor, only (2) and besides, the states are merely a derived notion, the primitive (phenomenologically given) notion being the qualities which correspond to the linear closed subspaces. . . .

By point (1), he means that a state $|\psi\rangle \in \mathcal{H}$, and a state $e^{i\theta}|\psi\rangle$ differing only by a phase, are physically equivalent. Point (2) takes as “phenomenologically given” special operators $\Pi : \mathcal{H} \rightarrow \mathcal{H}$ satisfying

$$\Pi^2 = \Pi, \quad \Pi^\dagger = \Pi.$$

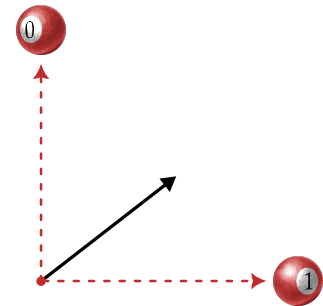
These are called *projection operators*, and the range $\mathcal{V}_\Pi = \Pi(\mathcal{H})$ of the operator is a closed subspace of \mathcal{H} . More physically, these projections correspond to binary “yes”/“no” measurements, in the sense that the eigenvalues are 0 (“no”) and 1 (“yes”).

This bears more than a little resemblance to classical bits and their algebraic realization. Indeed, early in his career, von Neumann pioneered the theory of *operator algebras*,²¹ where instead of studying the vectors transformed by operators like Π , we study the algebraic structure of the operators themselves. This led him organically to the logical aspects of the problem, and he collaborated with GARRET BIRKHOFF on the lattice-theoretic characterization of closed linear subspaces and many related problems. As Birkhoff put it, von Neumann's “brilliant mind blazed over lattice theory like a meteor”;²² a recurrent nova might be a better analogy. Ultimately, though, this “projective quantum logic” does not capture the true logical power of the quantum, and can be efficiently simulated on a classical computer.²³ But an expertise in operator theory and quantum logic would have been fertile ground when the right seed came along.

¹⁸ See “Über die Grundlagen der Quantenmechanik” (1927), Hilbert and von Neumann.

¹⁹ Loosely speaking, “closed” means that any point we can approach arbitrarily closely is contained in \mathcal{H} . The induced norm is simply $\|x\|^2 = \langle x, x \rangle$.

²⁰ Quoted in “Why John von Neumann did not Like the Hilbert Space Formalism of Quantum Mechanics (and What he Liked Instead)” (1996), Miklós Rédei.



Replacing states with projection operators, or equivalently, the 0 and 1 eigenspaces.

²¹ “On Rings of Operators I/II” (1936/7), Murray and von Neumann.

²² “Von Neumann and lattice theory” (1958), Garret Birkhoff.

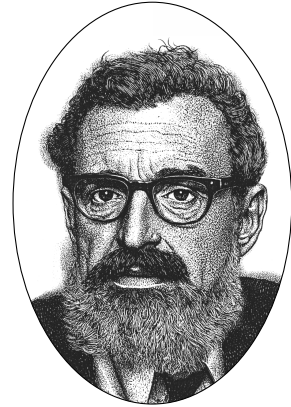
²³ “Nondeterministic testing of Sequential Quantum Logic propositions on a quantum computer” (2005), Matt Leifer.

If von Neumann skipped the Bikini Atoll tests to troubleshoot ENIAC, it's plausible that IRVING SEGAL, a promising young mathematician freshly decommissioned from the Ballistic Research Laboratory, would have heard about it. Before the Laboratory, Segal had worked on operator algebras with von Neumann at the IAS, where he had incidentally overlapped with Shannon. He was planning to return northeast to winter at Princeton and continue thinking about algebras and quantum mechanics, two topics von Neumann had mostly abandoned in favour of building thermonuclear weaponry.

Von Neumann's original work focused on the properties of the projections Π corresponding to binary measurements. In his view, these operators would be the "phenomenologically given" analogue of bits, rather than states like $|0\rangle$ and $|1\rangle$ in which he had "lost faith". The algebra of operators generated by these binary measurements is called a *von Neumann algebra*.²⁴ Like von Neumann, Segal wanted to capture quantum mechanics algebraically, but he had a few new desiderata. First, physics allows for measurements with richer outcomes than simply "yes" or "no". He was especially worried by the "mathematical difficulties in quantum electrodynamics,"²⁵ which Feynman was in the process of (non-rigorously) taming. Second, not every operator on Hilbert space should be allowed; we might not be able to measure it! Segal needed a formalism more expressive than von Neumann algebras, and more selective than Hilbert space.

He found it in the work of two Russian mathematicians, Gelfand and Naimark,²⁶ who were trying to understand algebraic structures called *normed rings*, where you can add, multiply, and measure length. They found a clever way to embed these rings as a set of operators on a Hilbert space. To Gelfand and Naimark, this was merely a technical bridge; to Segal, it was the royal road from algebra to quantum mechanics he had been seeking. The result of his meditations at Princeton was a paper that baptized the normed rings *C*-algebras*, connected them to quantum mechanics, and improved the embedding techniques. The method of identifying an abstract C*-algebra with a concrete set of operators on a Hilbert space is called the *Gelfand-Naimark-Segal (GNS) construction* in their honour.

To see how this algebra meets Segal's criteria, note that in the usual treatment of quantum mechanics, *any* self-adjoint linear operator $T = T^*$ on Hilbert space is an observable. A C*-algebra is much finer-grained; we include only the observables we care about, along with the things we can generate from them. A von Neumann algebra \mathcal{A} also turns out to be a special type of C*-algebra where, for any operator $T \in \mathcal{A}$ and eigenvalue λ of T , the projection $\Pi_{T,\lambda}$ onto the λ -eigenspace of T is in \mathcal{A} . But a general C*-algebra permits quantum reality to be richer than yes/no answers can exactly capture.

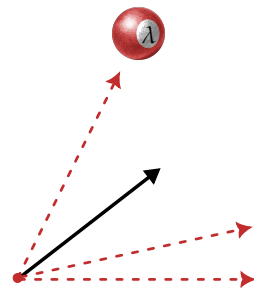


Irving Ezra Segal (1918–1998). *The mathematical prophet from the Bronx.*

²⁴ After his paper "Zur Algebra der Funktionaloperationen und Theorie der normalen Operatoren" (1930).

²⁵ "Irreducible representations of operator algebras" (1947).

²⁶ "On the embedding of normed rings into the ring of operators in Hilbert space" (1943), I. Gelfand, M. Naimark.



In a general C-algebra, we cannot reduce every measurement to its projections.*

Segal would make a career out of deep, unexpected connections between math and physics, helped in part by his stubborn and unyielding individuality. As his AMS obituary concludes,²⁷

... the full impact of the work of Irving Ezra Segal will become known only to future generations.

And in the words of John Baez:²⁸

Everyone who knows Segal will recall his inability to do things any way other than his own.

A short kid from the Bronx, Segal learned early to stand up for himself; a career in mathematical physics did not take the Bronx out of the boy. Despite his evident genius, von Neumann did not have Segal's depth,²⁹ though his breadth was unrivalled by any mathematician of the 20th century, perhaps history. If, by some cosmic glitch, Johnny was spared from the Baker test and guided towards computing with the quantum, deep might have converged with broad at Princeton, the winter of 1946, for a project of mutual interest.

4 *Through the looking glass*

Imagine the two in the faculty lounge at Old Fine Hall, pulling out a chalkboard to trade ideas; von Neumann with rapidfire cerebration, Segal his Bronx one-two of prickle and boldness. Behind them, the afternoon sun slants through stained-glass renderings of the uncertainty principle and relativity. Segal confides his goal of axiomatizing quantum mechanics with general normed rings, since, gesturing towards the windows, he suspects that "a relativistic continuum may not admit projections". Von Neumann asks some questions, then suggests in his offhand way a variant of Gelfand and Naimark's method of embedding the ring in Hilbert space, entirely ignorant of their results. Segal is wryly shocked, recovers, suggests improvements, and the two go on, developing C^* -algebras long into the evening.

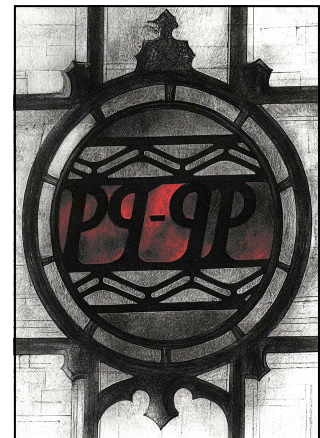
They continue over the next few days, fleshing out the representation theory and the rudiments of an axiomatic treatment of quantum fields. Von Neumann encounters a few little roadblocks, gets bored, then like the projectiles he has spent so much time on, launches himself once more: heading over to engineering to blueprint designs for an ENIAC successor, to Philadelphia to talk shop with Mauchly and Eckhert, and finally to Washington to sit on a nuclear advisory committee and nag the Weather Bureau for more money.

But the algebra won't leave him alone. Von Neumann goes to sleep one night thinking about ENIAC, programmability and the structure of cosmic rays... The next day, he drives in a state of cheerful mania

²⁷ "Irving Ezra Segal (1918–1998)" in *AMS Notices* (1999).

²⁸ *Ibid.*

²⁹ He was plagued by self-doubt, opining that he would be forgotten but "Gödel remembered with Pythagoras."



The left-hand side of Heisenberg's uncertainty principle, emblazoned in leadlight.

to Manhattan. The magician of Bell Labs receives him with courteous surprise, and they walk to a park overlooking the Hudson:³⁰

JVN: So, Shannon, you know the law of idempotency in a Boolean algebra, $x^2 = x$. [Shannon nods.] These variables are also commutative, in the sense that $x \cdot y = y \cdot x$, so your AND connective doesn't care about order of conjuncts. There's a noncommutative version of this where we replace Boolean variables by operators, and in particular we could build our theory around projectors obeying $\Pi^2 = \Pi$, so-called because they project vectors onto a subspace. [Shannon pauses briefly, then nods.] Good. Now, remember Irving, who was always disagreeing with Einstein? [A smile.] Well, he found a clever way to take rings of operators defined abstractly—like your Boolean algebra—and represent them as transformations of a Hilbert space. I want to understand if we can complete the parallelogram and build noncommutative circuits.

CES: I see. [Pause.] There is a perfect correspondence between the terms of a Boolean formula and the structure of the relay or switching circuit because, after a suitable identification of the components, physical laws of combination precisely match symbolic ones.

JVN: Yes yes, we need our algebraic laws to map onto the noncommutative laws obeyed by our wires. Enter quantum theory. Guided by my ancient work on orthocomplemented lattices, I have this [fishes napkin out of suit pocket] mildly soiled design schematic for inducing transitions in caesium. . . But what makes the circuits morally necessary?

CES: In what you call the “commutative” case, the hindrance of a complex circuit is determined by the hindrance of its components. [Long pause.] Hindrance means the passage of current, so I suppose we can view this as a law by which aspects of the physical state of the circuit are determined by the state of its components.

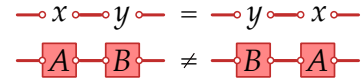
JVN: You mean “state” colloquially, but it's also a mathematical term of art for assigning numbers to operators, like current.

CES: Yes. I suppose a “state” here is a function assigning binary values to each independent propositional variable. Equivalently, this is an entry in a truth table. [Pause.] How is Segal's construction related?

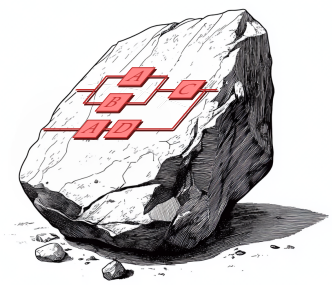
JVN: [Manic grin.] Ah, Shannon. Ah! The states of Segal's embedding are lines in the truth table. Irreducibility means restriction to the involved variables. . . But all this is trivial. Now, the question I pose to you, say Stibitz, Hamming of course, maybe Bardeen: can we build it?

Von Neumann leaves West Village with visions of “noncommutative architectures,” algebras as data and states as programs, his mind racing into the future. Shannon mentions the conversation to Mervin Kelly, director of research at Bell Labs, who thinks it over, heads to Princeton, and on his return quietly commissions a new research program. In branch T CrB, the rocks instructed us to replace hindrance with expectation, truth-tables with GNS, and Boolean algebras with functional analysis. This is not the branch we live in. But with a little imagination and some mathematical elbow grease, we can catch up.

³⁰ This dialogue, like the interaction with Segal, is fictitious.



Above, a commutative circuit where switches, corresponding to Boolean variables, can be interchanged. Below, a noncommutative circuit where switches are C^* -algebraic variables which cannot always be interchanged.



C^* -algebraic programming in silico, aka noncommutative circuits. These circuits do not exist, but might have if a cosmic ray struck ENIAC.

5 *A Westbeth postscript*

Visiting the old Bell Laboratories Building at 463 West Street, it's hard to detect the *genius loci* that moved it in former times. In the 60s, Bell moved its operations from Manhattan to a shiny new facility in New Jersey, and the West Street complex was converted into Westbeth Artists Community, the largest artist cooperative in the world. Its roster of luminaries includes photographer Diane Arbus, the choreographer Merce Cunningham, and puppeteer Ralph Lee, whose shambolic creatures still grace the commune in unexpected corners. It is more Coney Island than Wall Street.

After hours, I snuck in behind a departing resident and found myself in an interstitial maze of shoebox consultancies, potters' studios, office space, and miscellaneous storage, a *wunderkammer* of uninterpretable objects. . . I turned around and left before I could get irrevocably lost. Back in the courtyard, I understood that the spirit of Bell Labs remained alive in Westbeth: in its organized shaping of creative energies, its zest for human excellence, in scale and the network effects that come from so many unusual minds melting in the same pot. Shannon made theories in the same way that Lee made puppets: from an urge to make something beautiful, strange and new in the world, and from that urge alone.

The bricks have many stories to tell; Westbeth listens. Standing in the courtyard, I too strain to hear, and erect the puppets of von Neumann and Shannon a hundred yards away in the process. But there the whispers die; the rocks can only tell us so much. The work of building a noncommutative *ratiocinator*, the high-level *characteristica* to go with it, and a broader institutional culture of rock-listening—completing the arc of that proton over Philadelphia—is left to us.

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The mad puppets of Ralph Lee (1936–2023), oblique testament to the spirit of Bell Labs.

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